Mechanical Valves for On-Board Flow Control of Inflatable Robots

Lishuai Jin, Antonio Elia Forte, and Katia Bertoldi*

Inflatable robots are becoming increasingly popular, especially in applications where safe interactions are a priority. However, designing multifunctional robots that can operate with a single pressure input is challenging. A potential solution is to couple inflatables with passive valves that can harness the flow characteristics to create functionality. In this study, simple, easy to fabricate, lightweight, and inexpensive mechanical valves are presented that harness viscous flow and snapping arch principles. The mechanical valves can be fully integrated on-board, enabling the control of the incoming airflow to realize multifunctional robots that operate with a single pressure input, with no need for electronic components, cables, or wires. By means of three robotic demos and guided by a numerical model, the capabilities of the valves are demonstrated and optimal input profiles are identified to achieve prescribed functionalities. The study enriches the array of available mechanical valves for inflatable robots and enables new strategies to realize multifunctional robots with on-board flow control.

1. Introduction

From minimally invasive surgical tools[1–4] and assistive devices,[5–8] to compliant grippers[9–12] and video game add-ons[13–15], inflatable soft robots have claimed an entire domain of applications for which safe interactions with the surrounding environment is the priority.[16–19] They are inherently compliant, easy to fabricate, and able to achieve complex motions harnessing the input pressure. However, the control of the fluid flow typically requires complex infrastructures comprising power sources, solenoid valves, electronic circuits, and pumps. As such, the development of strategies for an efficient actuation and control of inflatable soft robots is essential to the advancement of the field. Toward this end, various principles have been investigated to achieve flow control through pneumatic valves: flapping membranes inside the airways,[20–22] collapsing soft channels,[23–27] and kinks in soft tubes.[28,29] In addition, the snapping of bistable elastomeric membranes has recently been employed to design soft valves that use a second pressure signal to switch between states[28,30–32]—the first of their kind to be built entirely from soft materials and to operate without the need of electronics. Such valves have demonstrated to provide a platform for the design of logic elements,[30] ring oscillators that induce periodic motion using constant-pressure,[31] as well as electronics-free pneumatic circuits for controlling soft-legged robots.[32]

The design of mechanical valves that can be integrated into soft robots and operated without the need of additional inputs is, however, still at an early stage. A more extensive library of elements would be ideal in order to expand the capabilities of soft robots that can operate without the need of electronic components.

Inspired by the recent progress in the design of mechanical valves, here we use different strategies to design and realize simple, easy to fabricate, lightweight and inexpensive mechanical valves, that are driven by the same pressure input used for the robot’s actuation. Our designs harness both viscous flow and snapping arch principles, can be fully integrated on-board and enable the control of the incoming airflow to realize multifunctional robots that operate with a single pressure input and no need for electronic components. These include a soft robotic arm capable of achieving multiple trajectories, a robot capable of climbing vertically in a tube carrying two times its own weight and even grasping an object and pulling it down, and a rolling robot that can successfully navigate in two directions.

2. Our Mechanical Valves

To simplify the flow control in soft robots, we design, realize, and test four fluidic mechanical valves. The first design, which we refer to as viscous valve, uses a narrow tube with length $l_{tube} = 25$ mm and internal diameter $d_{tube} = 0.21$ mm to provide a transient pressure difference between adjacent actuators (Figure 1a) and causes their sequential activation. To demonstrate the capability of this valve we place it between a rigid chamber and...
The chamber with air pressurized at $p_{\text{input}}$ = 10 kPa for $t_{\text{input}}$ = 120 s and monitor the pressure evolution inside the soft actuator, $p_{\text{output}}$. As shown in Figure 1e, the valve introduces a transient pressure difference between the chamber and the actuator and that it takes ≈100 s for $p_{\text{output}}$ to stabilize to $p_{\text{input}}$ during both inflation and deflation (see Sections S1.1, S1.2, and S2.1, Supporting Information). Next, we connect the center of the buckled plate to a 3D-printed piston, with head diameter $d_{\text{head}}$ = 5 mm). This arch remains curved upwards for $t_{\text{input}}$ = 20 kPa. A as a consequence, the piston travels back across the notch, turning the valve off. This results in a steady pressure offset between the inlet and outlet. Even after the input pressure is removed, the pressure inside the kirigami actuator is kept at such steady level by the valve, so that the actuator remains inflated.

Further, by simply varying the mounting angle $\theta_{\text{plate}}$ to 0° one can transform the hysteretic valve into a bistable one, which switches state and remains open even when the input pressure is removed. To realize such bistable valve and have it operating at around 20 kPa, we reduce the thickness of the metallic plate to $t_{\text{plate}}$ = 0.05 mm (while keeping $w_{\text{plate}}$ = 17.5 mm and $t_{\text{plate}}$ = 5 mm). This arch remains curved upwards for $p_{\text{input}}$ < 20 kPa. Therefore, for low pressure inputs the measured outlet pressure

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**Figure 1.** Design and characterization of the mechanical valves. a–c) Schematics and experimental snapshots of the valves. a) The viscous valve (indicated by the blue icon) consists of an acrylic plate and a narrow tube with length $l_{\text{tube}}$ = 25 mm and inner diameter $d_{\text{tube}}$ = 0.21 mm. b) The one-way viscous valve (indicated by the purple icon) is a viscous valve with an additional one-way gate which cause a pressure lag only in one direction. c) The hysteretic (indicated by the red icon) and bistable (indicated by the yellow icon) valves exhibit tunable mechanical responses controlled by a snapping arch connected to a movable piston. d) Experimental setup to test the valves. e–h) Pressure evolution at the inlet and outlet of each valve. e) Viscous valve: a delay in the output response in respect to the input is visible in inflation. f) One-way viscous valve: a delay in the output response in respect to the input is visible only in inflation. g) Hysteretic valve: the on/off thresholds for this valve are ≈20 kPa and ≈2 kPa, respectively. h) The bistable valve only needs energy to switch its on/off state and would maintain that state once it actuated. Scale bars = 15 mm.
is equal to zero (see results in Figure 1h for $p_{\text{input}} = 10$ kPa). However, if the input pressure goes over $p_{\text{input}} = 20$ kPa, the arch snaps to its inverted stable state and the valve opens (see dark orange line in Figure 1h). Importantly, our bistable valve remains in the opened state even after the input pressure is removed, resulting in a synchronous pressure variation between the input and output (i.e., once the input is removed, the output drops to zero at the same time). A large negative pressure impulse is needed to close this valve (see Figure S14, Supporting Information for additional details about the effect of geometric parameters on the snapping response of our hysteretic and bistable valves).

3. Robotic Arm with Different Trajectories

After introducing our four mechanical designs, we now show how to integrate them with several actuators and realize multifunctional robotic systems powered by a single pressure input.

As a first example, we combine two bending kirigami actuators and a viscous valve to realize a robotic arm that can assume different trajectories by varying a single pressure input. As a reference, in the case of the two actuators simply connected without a valve in between, the robotic arm starts straight and morphs into an S-shape upon application of a rectangular pressure pulse ($p_{\text{input}} = 20$ kPa for $t_{\text{input}} = 20$ s, see inset in Figure 2a). The actuator’s tip reaches a point 102 mm away from the initial position (I in Figure 2a) and goes back to its initial configuration when the pressure returns to 1 atm (Figure 2a). Clearly, for such robotic arm the deformation is only affected by the magnitude of $p_{\text{input}}$.

However, by introducing a viscous valve between the two actuators we can control the trajectory of the tip by modulating the pressure-time profile in input (see Figure 2b–d). To demonstrate the concept, we power the new version of the robotic arm with a short rectangular pressure pulse ($p_{\text{input}} = 20$ kPa for $t_{\text{input}} = 4$ s) and immediately back to the initial position when the pressure is removed. c) When a longer rectangular pressure pulse is fed to the robot ($p_{\text{input}} = 20$ kPa for $t_{\text{input}} = 26$ s) the end point of the robotic arm is able to reach three different positions. d) When undergoing a gradually changed pressure, the robotic arm deforms to an S-shape, resembling the same deformation of the arm without valve. Scale bars = 30 mm.
$t_{\text{input}} = 4$ s, see the inset of Figure 2b). This results in the top actuator deforming instantaneously upon pressurization, whereas the presence of the viscous valve delays the activation of the bottom one. Therefore, the endpoint of this robotic arm moves further away compared to the previous version when pressurized (reaching position II, 196 mm away from the initial position) and immediately back to the initial position when the pressure is removed. If a longer rectangular pressure pulse is fed to the robot ($p_{\text{input}} = 20$ kPa for $t_{\text{input}} = 26$ s, see the inset of Figure 2c), the end point of the robotic arm is still able to reach position II during the first few seconds of the cycle. However, as time increases, the bottom actuator also bends, so that the robotic arm gradually mutates into the S-shape configuration, and the endpoint moves to position I. Further, when the input pressure is removed (i.e., for $t_{\text{input}} > 26$s), the top and bottom actuators cannot deflate simultaneously due to the presence of the valve. Consequently, when the top actuator goes back into the initial, straight position, the bottom one is still bent: the endpoint reaches a new position III (65 mm apart from position O) on the left side of the initial configuration before returning to the rest position. Finally, a similar trajectory to that obtained in the absence of the viscous valve can be achieved, by gradually varying the pressure ($p_{\text{input}}$ increases from 0 to 20 kPa in 50 s and then decreases to 0 kPa in 100 s—Figure 2d). Note that in this case a left swing is observed at the end of the cycle. This is due to the step-like nature of the pressure input, and can be eliminated by using a smoother pressure input (see Sections S1.3 and S2.3, Supporting Information for additional details about fabrication and testing of the robotic arm).

Whereas in Figure 2 we consider a robotic arm with a viscous valve, the range of achievable trajectories can be further enlarged by incorporating a one-way viscous valve (Figure S17, Supporting Information) and connecting more than two actuators (Figures S18 and S19, Supporting Information). Finally, the behavior of the robotic arm under different pressure inputs can be nicely captured by a simple numerical model based on the Navier–Stokes equations \cite{14} expanded to take into account air compressibility (see Section S3.1, Supporting Information for additional details about the model).

4. Tube Climbing Robot

Thus far, we have demonstrated the capabilities of individual valves (viscous and one-way viscous valve) for the realization of a robotic arm with tip trajectories that can be prescribed by modulating the input pressure. As second demonstration we combine different valves together and realize a robot capable of climbing vertically in a pipe. Moreover, the robot can carry twice its own weight and even grasp an object. Differently from previous tube-climbing robots\cite{15–38} our system operates with a single pressure input.

To realize such robot we connect two expanding kirigami actuators to an extending one via a one-way viscous valve and a viscous valve (Figure 3a). Upon pressurization, the bottom actuator expands instantaneously and grips onto the pipe (Figure 3b). At the same time the top and middle actuators deform gradually due to the presence of the viscous valves. This results in the head of the robot moving upwards before the top actuator is able to grip onto the pipe. As the pressure input is removed, the top actuator remains fully inflated, due to the lag introduced by the viscous valve, and acts as anchoring point. Differently, due to the presence of the one-way viscous valve which offers negligible resistance for the air to instantaneously escape, the middle and bottom actuators deflate synchronously. With the top actuator still anchored, the deflation translates in an upward movement for the lower part of the robot, which closes the loop.

Note that such deformation sequence is promising toward enabling a climbing motion, since it is asymmetric and makes the robot extend upwards. However, it only translates into vertical climbing if the frictional force between the inflated top actuator and the tube is i) larger than the weight of the robot (or it will fall) and ii) smaller than the axial extension force generated by the middle actuator (or it will not extend upwards). Since the friction and extension force are dependent on the pressure in each actuator, a tailored input pressure signal is crucial for the ability of the robot to climb. In order to identify input signals that enable climbing, we first measure i) the evolution of the friction between the pipe and the expanding actuator and ii) the extension force generated by the middle actuator as a function of the internal pressure. As shown in Figure 3c, we find that the minimum pressure inside the expanding top actuator should be larger than $p_{\text{low}} = 2.5$ kPa to prevent slipping and smaller than $p_{\text{high}} = 4.2$ kPa to enable axial extension toward the top.

To identify a suitable input signal that satisfy these requirements, we run 77 numerical analysis where we systematically vary $p_{\text{input}}$ and $t_{\text{input}}$ (with $p_{\text{input}} \in [7, 19]$ kPa and $t_{\text{input}} \in [38, 58]$ s) while keeping the actuation period equal to $T = 60$ s. In Figure 3d, we report the minimum pressure recorded for the top actuator after the first cycle, $p_{\text{min}}$, as a function of $p_{\text{input}}$ and $t_{\text{input}}$. Within the explored design space we find a “climbing” region (bounded by the two dashed lines) for which $p_{\text{min}}$ is greater than $p_{\text{low}}$ and smaller than $p_{\text{high}}$. To validate these numerical predictions, we conduct experiments in which we provide an input pressure of $p_{\text{input}} = 13$ kPa and select $t_{\text{input}}$ guided by the results of Figure 3d. Note that in order to maximize the extension toward the top of the middle actuator and, in turn, increase the efficiency of the robot, for a given value of $p_{\text{input}}$, $t_{\text{input}}$ should be selected to be as close as possible to the lower boundary of the “climbing” region. In fact, the closer we operate to the lower boundary, the greater is the difference between the vertical extension force exerted by the middle actuator and the friction between top actuator and the tube (Figure 3c, red segment). As such, we first choose $t_{\text{input}} = 45$ s (Figure S20a, Supporting Information). However, because of small discrepancies between our experiments and model (which slightly over predicts the minimum pressure in the top actuator—Figure 3e), we find that the frictional force is not enough to prevent falling. Therefore, we increase $t_{\text{input}}$ to 48 s to move slightly away from the lower boundary of the domain and find that for this input the robot climbs upwards and moves of 13 cm in 25 cycles (Figure 3f). In Figure 3e, we report the resultant input pressure profile fed into the bottom actuator of the robot (green curve), along with the pressure profiles measured in all the chambers (continuous lines) and those predicted by the model (dashed lines) and find very good agreement between the two sets of data. Further, these results are in agreement with the predicted climbing sequence shown in Figure 3b. In fact, when $p_{\text{input}}$ is removed, the pressure decreases in all three actuators. However, while this drop is almost instantaneous in the
Figure 3. Multifunctional climbing robot. a) Schematic of the climbing robot comprising three kirigami actuators (two expanding and one extending actuator), a one-way viscous valve and a viscous valve. b) Climbing of the robot is enabled by the sequential inflation and deflation of the actuators, which can be modulated by varying the pressure-time profile of a single pressure input. c) The evolution of the friction between the pipe and the expanding actuator as well as the extension force generated by the middle actuator, as a function of the internal pressure. According to the plots, the minimum pressure inside the expanding top actuator should be larger than $p_{\text{low}} = 2.5 \text{ kPa}$ to prevent slipping and smaller than $p_{\text{high}} = 4.2 \text{ kPa}$ to enable axial extension toward the top. Error bars indicate the standard deviation for the measurements. d) Phase diagram of the minimum pressure inside the top actuator (after the first cycle). The area of the diagram that is bounded by the two dashed lines identifies the input parameters for which the robot will achieve climbing. e) Comparison between the experimental and numerical results of the pressure evolution inside the actuators. The good agreement between them enables one to predict the behavior of the robot using the numerical model. f) Experimental demonstration of the climbing robot which climbs 13 cm in 25 cycles with $p_{\text{input}} = 13 \text{ kPa}$, $t_{\text{input}} = 48 \text{ s}$ and $T = 60 \text{ s}$. g) Schematic of the climbing robot capable of grasping an object and pulling it down, which is realized by adding an additional hysteretic valve and gripper to the top actuator. h) Experimental snapshots of the climbing robot with the gripper. Scale bars = 30 mm.

Bottom and middle actuator, it is less sharp in the top one due to the delay introduced by the viscous valve. Importantly, this results in a non-zero minimum pressure for the top actuator, which enables upward retraction of the other two chambers while assuring anchoring. Additionally, it is worth noticing that if $t_{\text{input}}$ is further increased to 55 s, the robot does not move upwards as the frictional force between the top actuator and the pipe becomes too large (Figure S20b, Supporting Information). Further, we note that, using the same approach, the input signal can be also optimized to make the robot carrying two time its weight...
Figure 4. Rolling robot. a) A hexagonal rolling robot integrated with a modified version of the bistable valve and six viscous valves is able to roll bi-directionally with a single pressure input. The bistable valve has two states, which allow it to dispense the input flow into two separate circuits, each connecting six chambers (highlighted in white and orange). Each circuit has three chambers connected directly to the bistable valve, while other three chambers are connected to the bistable valve through viscous valves. b) Experimental snapshots of the rolling robot. c) Operating principle of the rolling robot. The chambers on the rolling robot inflate/deflate sequentially due to the presence of viscous valves pushing the robot away from one face to another. The bistable valve, which can be switched by applying a pressure burst, dispenses the input flow into different circuits enabling the bi-directional rolling. d) Experimental snapshots of the robot rolling in both directions. Scale bars = 50 mm.

(Figure S20d and Movie S3, Supporting Information). Finally, we want to emphasize that the slow speed of the robot is due to the low flow rate of our input pump (ITV1030 by SMC Cooperation, with flow rate around 7.5 mL s$^{-1}$ at pressure of 20 kPa) rather than our mechanical valves and that the speed could be largely increased by using a better input pump to increase the flow rate (Figure S22, Supporting Information).

Next, to enhance the functionality of the robot and making it capable of grasping an object and pulling it down, we add a grip- per to its head (comprising two PneuNets$^{27}$) and connect it to the top actuator via an hysteretic valve (Figure 3g). Since the valve is embedded into the top actuator, it affects the actuator’s pressure-volume relationship and, in turn, alters the input parameters required for climbing. Therefore we rerun the numerical analysis accounting for the internal volume reduction of the top actuator and find that larger values of $p_{\text{input}}$ are required to achieve locomotion (Figure S21, Supporting Information). Guided by the numerical analyses we then choose $p_{\text{input}} = 15$ kPa and $t_{\text{input}} = 48$ s. As shown in Figure 3h, we find that, when $p_{\text{input}} = 15$ kPa is cyclically applied, the robot navigates upwards toward the object. Then, as the robot approaches the object, a pressure higher than the threshold pressure of the hysteretic valve ($p_{\text{input}} = 22$ kPa) is supplied (the input pressure profile for the experiments is reported in the bottom panel of Figure 3h). The valve opens and actuate the gripper, which grasps the object in the pipe. Afterwards, when the pressure in the gripper approaches the pressure inside the top actuator, the hysteretic valve snap back, switching off. At this point, the gripper remains inflated, keeping hold of the object. Finally, the three kirigami actuators can be deflated to remove the anchoring points and let the robot slide down with the captured object.

5. Rolling Robot

To further illustrate the capability of our valves, we design and test a hexagonal rolling robot integrated with one bistable valve and six viscous valves. This robot is different from other rolling robots$^{31,39–42}$ due to its ability to roll bi-directionally with a single pressure input. The robot comprises twelve inflatable chambers evenly distributed along the perimeter of an hexagon (two on each side, see Figure 4a,b). Such chambers act as legs and by selectively inflating and deflating them we can push the robot away from a stable equilibrium and make it roll forward or backward.

Such rolling motion can be realized with a single input pressure by introducing viscous valves. More specifically, we form a circuit connecting six inflatable chambers and the input source (white connections in Figure 4a). Three of these chambers are connected directly to the pressure supplier. Alternating these, there are three additional chambers, each of which is connected to a viscous valves before being connected to input
(see schematics in Figure 4a). When a pressure input is supplied, the chambers directly attached to the input valve inflate first (snapshot 2 in Figure 4c). This unbalances the robot to the point that it rolls over the hexagon’s corner and lay flat on an edge where the chamber is still deflated (snapshot 3 in Figure 4c). At this point the chambers attached to the viscous valves, which suffer an inflation delay, start to inflate too (snapshot 4 in Figure 4c). When the input pressure is removed, the chambers with no viscous valve deflate instantaneously, while the ones with viscous valves keep the inflated state temporarily and push the robot to rotate further (snapshot 5 in Figure 4c) until all the chamber deflates to the initial state (snapshot 6 in Figure 4c). The robot can keep rotating forward by repeating the aforementioned procedure. Note that, in an effort to speed up the locomotion, in this demonstration the diameter of the viscous valves’s narrow tube was increased to $d_{\text{tube}}=0.26$ mm.

Further, the direction of rolling can be changed by introducing an identical circuit that connects the remaining six chambers and a slightly modified design of the bistable valve in which the notch is replaced with a hole with diameter of 1.6 mm. When the input source is connected to this hole (see schematics in Figure 4a), the new version of the bistable valve is able to switch the flow from one circuit to the other (highlighted in white and orange in Figure 4a). This is possible by applying a negative pressure burst to the input to snap the arch and, in turn, change the position of the piston. The piston travels over the hole and switches the valve’s output to the second circuit (orange tubes), where the inflatable chambers are positioned in the opposite order. The robot can then start rotating in the opposite direction by repeating the same inflation cycle (snapshots 7–12 in Figure 4c).

To demonstrate the concept experimentally we fabricate a prototype out of elastomeric materials by means of a molding approach. From the experimental snapshots in Figure 4d, we can see that the robot rolls from one face of the hexagonal frame to the next as the chamber on each face sequentially inflate and deflate. Further, by applying a negative pressure pulse ($p_{\text{burst}} = -25$ kPa, see Figure S11 and Movie S4, Supporting Information) we can switch the state of the bistable valve and change the direction of rolling.

6. Conclusion

In summary, we have designed and built simple, easy to fabricate, lightweight, and inexpensive mechanical valves that can be easily integrated with soft actuators to control the airflow and realize robotic systems that operate with a single pressure input. Our mechanical valves provide new opportunities to realize sequential operation robots, and enrich the array of available valves with new designs harnessing instabilities in compliant shell elements.

Supporting Information

Supporting Information is available from the Wiley Online Library or from the author.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Keywords

climbing, mechanical valves, robotic arm, rolling, soft robots

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- Movies S1 to S4
1. Fabrication

In this Section we provide details on the fabrication of our mechanical valves as well as the three robotic systems (i.e. the robotic arm, the climbing robot and the rolling robot).

1.1. Mechanical valves. To simplify the flow control in soft robots, we introduce four fluidic mechanical valves: (i) viscous valve, (ii) one-way viscous valve, (iii) hysteretic valve and (iv) bistable valve.

Fig. S1. Design of the mechanical valves. Experimental snapshots of (A) viscous valve, (B) one-way viscous valve and (C) hysteretic or bistable valve.

- **Viscous valve**: The viscous valve tested in this study comprises a circular acrylic plate with embedded a hollow tube with small diameter (Fig. S1A). Specifically, our viscous valves are fabricated using the following steps

  - Laser cut a circular acrylic plate with diameter of 30 mm and thickness of 1/16 inch.
  - Cut two holes in the acrylic plate: (a) one at the center with diameter of 3 mm that is used to connect adjacent actuators via a bolt; and (b) one with smaller diameter of \( \sim 0.8 \) mm that is used to embed the narrow tube.
  - Insert a narrow tube through the hole b of the acrylic plate and seal the gap between the tube and the hole using glue (Sil-Poxy Silicone Adhesive, Smooth-On). The narrow tubes used in this study are cut from glue needles (FEITA) and have length of 25 mm and inner diameter varying from 0.16 mm to 0.51 mm.

- **One-way viscous valve**: To obtain a valve with unidirectional pressure drop, we add a one-way valve to the viscous valve described above. Our one-way valves are fabricated using the following steps (Fig. S1B):

  - Cut another hole (with diameter of 3 mm) on the acrylic plate of the viscous valve (denoted with c in Fig. S1B). To avoid conflict when assembling the valve with actuators, this hole is in a diametrically opposite position to the one used for the needle.
  - Cast a rectangular shape membrane using Ecoflex 00-50 with length of \( \sim 10 \) mm, width of \( \sim 6 \) mm and thickness of \( \sim 1 \) mm.
  - Glue the two short edges of the membrane on the acrylic plate to cover hole c. If the membrane is attached to the top of the plate, it allows only flow from the bottom to the top. In fact the flow bends the membrane and allows the air through hole c.
Please note that when the valves are integrated with the actuators, the hole in the center of the acrylic plate should be sealed by an appropriate sealing ring to avoid air leaking (see Fig. S3 and S4).

- **Hysteretic valve:** Our hysteretic valve exploits the snapping of elastic arches (Fig. S1C). To fabricate the valve, we follow the steps below:
  - Laser cut rectangular plates with width $w_{\text{plate}} = 5$ mm and length $l_{\text{plate}} = 17.5$ mm out of 1095 spring steel shims (McMaster-Carr product ID: 9503K31) with thickness $t_{\text{plate}} = 0.075$ mm and Young’s modulus $E = 170$ GPa.
  - Use an Ultimaker S3 3D printer to fabricate an holder out of PLA material with two slits oriented at an angle $\theta_{\text{plate}} = 45^\circ$ with respect to the horizontal direction.
  - Apply an axial force to buckle the rectangular plate into an arch and slide its ends into the two slits of the 3D-printed holder and glue the arch to the slits using ethyl 2-cyanoacrylate glue (Krazy Glue, NC).
  - 3D print a piston and glue the piston at the center of the arch using Sil-Poxy (Smooth-On).
  - Cut a segment with length of $\sim 40$ mm out of a 10 ml plastic syringe with inner diameter of 22 mm. This will form the chamber for the valve.
  - Cut a dome-shaped notch (with depth around 1 mm and diameter around 5 mm) on the internal surface of the cylindrical segment using a rotary tool workstation (220-01 Dremel). When the piston moves across the notch, the on/off state of the valve is changed.
  - Mount O-rings (Oil-Resistant Soft Buna-N, McMaster-Carr product ID: 2418T138) on the piston and the holder to prevent leaking between the piston (or holder) and the chamber.
  - Insert the construction of holder, arch and piston into the chamber.

- **Bistable valve:** The bistable valve is fabricated following the same steps used for the hysteretic valve. The only differences between the two designs are the thickness of the plate (for the bistable valve we use a metallic plate with thickness $t_{\text{plate}} = 0.05$ mm) and the mounting angle $\theta_{\text{plate}}$ (for the bistable valve we use $\theta_{\text{plate}} = 0^\circ$).
1.2. **Kirigami actuators.** To realize the robotic arm and climbing robot shown in Fig. 2 and 3 of the main text, we use cylindrical kirigami actuators fabricated following the procedure described in our previous work (1) - the only difference is that here we add another thin layer (thickness $\sim 1$ mm) of Ecoflex 00-50 inside the actuator to enhance its robustness and durability. Specifically, we use three types of actuators which support bending (Actuator I in Fig. S2), extension (Actuator II in Fig. S2) and expansion (Actuator III in Fig. S2). The geometric parameters defining each actuator are shown in Fig. S2 and listed in Table S1. Note that Actuators II and III comprise identical units for the entire structure, whereas to enable bending in Actuator I we introduce one column of unit cells with increased ligament size $\delta_2$ (shown as purple units in Fig. S2).

![Fig. S2. Kirigami actuators. Three types of kirigami actuators are used in realizing bending (Actuator I), extension (Actuator II) and expansion (Actuator III) deformation. The geometric parameters of the actuators are listed in Table S1.](image)

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<th>Geometric parameters defining our kirigami actuators. All parameters are defined in Fig. S2.</th>
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1.3. **Robotic arm.** The robotic arm is constructed by connecting two bending kirigami actuators (Actuator I in Table S1 and Fig. S2) through a viscous valve or a one-way viscous valve. Specifically, we fabricate the robotic arm using the following steps:

- Fabricate two identical kirigami actuators (Actuator I in Table S1 and Fig. S2) following the procedure described in our previous work (1). The only difference is that here we add another thin layer (thickness \( t \sim 1 \text{ mm} \)) of Ecoflex 00-50 inside the actuator to enhance its robustness and durability. More specifically, we pour additional Ecoflex inside the kirigami actuator and rotate it slowly to achieve a uniform coating.

- Glue perforated acrylic plates at the ends of the actuator to facilitate connection with other actuators (see Fig. S3). Further, we 3D print a mold to cast elastomeric layers with thickness \( \sim 1 \text{ mm} \) and the same shape as the acrylic plates. Such elastomeric layers are placed between the acrylic plates and the valves to avoid leakages (see Fig. S3).

- Arrange the two identical bending actuators so that the columns of unit cells with increased ligament size \( \delta_2 \) are diametrically opposed (Fig. S3).

- Introduce a viscous valve (or one-way viscous valve) between the two actuators to regulate the flow between them (as shown in Fig. S3). Secure the connection between the actuators and the valve by threading a screw through the acrylic plates and the hole places in the center of the valve.

- Connect a tube to the top actuator to provide the pressure input.

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**Fig. S3.** Schematic of the robotic arm. The robotic arm is constructed by connecting two bending kirigami actuators (Actuator I in Table S1 and Fig. S2) separated by a viscous valve or one-way viscous valve.
1.4. Tube climbing robot. As shown in Fig. S4A our climbing robot comprises two expanding kirigami actuators (Actuator III in Table S1) and an extending one (Actuator II in Table S1) connected via a one-way viscous valve and a viscous valve. Further, to enable grasping, a hysteretic valve and a gripper consisting of two PneuNets (2) is added at the top (Fig. S4B). To fabricate the robot, we follow the steps below:

• Fabricate two identical expanding kirigami actuators (Actuator III in Table S1 and Fig. S2) and one extending actuator (Actuator II in Table S1 and Fig. S2) following the procedure described in our previous work (1). The only difference is that here we add another thin layer (thickness $t \sim 1$ mm) of Ecoflex 00-50 inside the actuator to enhance its robustness and durability. More specifically, we pour additional Ecoflex inside the kirigami actuator and rotate it slowly to achieve a uniform coating.

• Glue perforated acrylic plates at the ends of the actuators to facilitate connection with other actuators (see Fig. S4A). Further, we 3D print a mold to cast elastomeric layers with thickness $\sim 1$ mm and the same shape as the acrylic plates. Such elastomeric layers are placed between the acrylic plates and the valves to avoid leakages (see Fig. S4A).

• Introduce a viscous valve between the top extending actuator and the elongating one and a one-way viscous valve between the bottom extending actuator and the elongating one (as shown in Fig. S4). Secure the connection between the actuators and the valve by threading a screw through the acrylic plates and a hole places in the center of the valve.

• Connect a tube to the bottom actuator to provide the pressure input.

Additionally, to realize a robot capable of grasping an object, the following steps are also needed:

• 3D print the two-part mold shown in Fig. S5A. This mold enables casting of PneuNets with length $l_{\text{gripper}} = 30$ mm, width $w_{\text{gripper}} = 10$ mm and thickness $t_{\text{gripper}} = 5$ mm.

• Prepare Ecoflex 00-30 and Ecoflex 00-50 to cast the PneuNets.

• Pour Ecoflex 00-30 inside the main body mold; pour Ecoflex 00-50 inside the base mold up to $2/3$ of its height ($\sim 2$ mm, Fig. S5B).

• Cure the Ecoflex for about an hour at room temperature.

• Remove the cured Ecoflex 00-30 from the main mold

• Apply a layer (with thickness of $\sim 0.5$ mm) of uncured Ecoflex 00-50 on top of the cured Ecoflex 00-50 (while keeping the part in the base mold)

• Slowly place the Ecoflex 00-30 structure obtained from the main mold on top of the base mold. The uncured Ecoflex 00-50 enables bonding between the two pieces.

• Remove the soft actuator from the mold.

• Insert a short tube in one end of the PneuNets.

• Laser cut a circular acrylic plate with diameter of $30$ mm and thickness of 1/16 inch.
• Cut three holes (with diameter of 3 mm) in the acrylic plate: (a) the one at the center is used for the connection with the top actuator via a screw; and (b) the other two holes, located at the margin of the plate, are used to pass through the short tubes that connect to the Pneunets.

• Apply glue (Sil-Poxy Silicone Adhesive, Smooth-On) to secure the tube and Pneunets to the acrylic plate.

• Attach a hysteretic valve to the top expanding actuator and connect the gripper to its chamber. Apply glue (Sil-Poxy Silicone Adhesive, Smooth-On) to fix and seal the valve and the gripper.

• The robot is ready to be tested.
1.5. Rolling robot. The rolling robot demonstrated in this study is made of nearly incompressible silicon rubbers. Specifically, we use Elite Double 32 (with green color and initial shear modulus $\mu = 0.35$ MPa, Zhermack) for the hexagonal frame of the robot and Ecoflex 00-50 (with white translucent color and initial shear modulus $\mu = 0.0405$ MPa, Smooth-On, PA) for the deformable chambers. 12 chambers are evenly distributed along the perimeter of the robot (two on each side), which are connected to the pressure source via six viscous valves and a modified bistable valve. The robot is fabricated using a molding approach according to the following 8 steps (Fig. S6):

- **Step A-1:** 3D print the mold shown in Fig. S6A using PLA material with a Ultimaker S3 printer. We insert screws inside the mold to facilitate the connection of the tubes in *Step G*.
- **Step A-2:** pour Ecoflex 00-50 into the mold to cast the skeleton of the chambers and let it cure for 30 minutes at room temperature. Note that the height of the Ecoflex should not be higher than that of the mold’s islands shown in Fig. S6A.
- **Step B:** pour Elite Double 32 into the mold (on top of the cured Ecoflex 00-50) to cast one side of the frame.
- **Step C:** wait for Elite Double 32 to cure for 30 minutes.
- **Step D:** remove the cured Ecoflex 00-50 and Elite Double 32 from the mold. Note that these two materials bond naturally.
- **Step E:** use another mold to cast the other hexagonal frame of the robot, pour Elite Double 32 into the mold until the thickness is about $4/5$ of the mold’s thickness.
- **Step F:** when the Elite Double 32 frame is half cured (after $\sim15$ minutes), pour more Elite Double 32 onto the half cured skeleton (to reach the full thickness) and then place the structure from *Step d* on top of it to seal the frame and the chambers.
- **Step G:** connect two sets of tubes to the robot to form two circuits. Each set has six tubes attached to the six chambers (one at each side of the robot). Three tubes in each set are equipped with a viscous valve. Note that to facilitate the (dis)assembling of the valves from the robot, we split each tube into two segments and attach them using a luer lock (with internal diameter 1/8 inch). We then insert the viscous valves into the luer lock. Note that in this case, we glue and seal the narrow needle (with length $l_{tube} = 25$ mm and diameter $d_{tube} = 0.26$ mm) of the viscous valve directly into the luer lock using Sil-poxy).
- **Step H:** Connect the two sets of tubes via a bistable valve (see the connection in Fig. S7).

- Modify the design of the bistable valve to enable bi-directional rolling. More specifically, we replace the notch with a hole with diameter of 1.6 mm. When the input source is connected to this hole (Fig. S7A), the new version of the bistable valve is able to switch the flow from one circuit to the other. This is possible by applying a negative pressure burst to the input to change the position of the piston and snap the arch at the same time, which forces the piston to stay in the new position. The connection between the bistable valve and viscous valves is shown in Fig. S7B.
- Add additional acrylic rods to one face of the robot to stabilize it and prevent it to fall sideways during the test (see Fig. 4D in main text).
Fig. S6. Fabrication of the rolling robot. Snapshots of the 8 steps required to fabricate the robot.
Fig. S7. Connection of the valves for rolling robot. (A) Working principle of the modified bistable valve. By changing the position of the piston, we can switch the flow from one circuit to the other. (B) The connection between the bistable valve and the two circuits.
2. Experiments

In order to characterize the effect of the valves on the response of the robotic systems, we use pressurized air of the order of a few kPa. To achieve such levels of pressure, we decrease the pressure from the wall air-outlet (∼200 psi) using two pressure regulators (B74G-4AK-AD3-RMN by IMI Norgren Inc and ITV1030 by SMC Cooperation). The first pressure regulator reduces the inlet pressure from 200 psi to about ∼200 kPa, while the second one accurately controls the pressure in the range [0, 200] kPa. Further, to turn on and off the input pressure we used a standard two-way solenoid valves (KVE32PL24FF by Kamoer) to switch on/off the inlet of the system to the atmosphere, so that the actuators could deflate and return to their initial state.

2.1. Characterization of the mechanical valves.

To characterize all our mechanical valves, we connect each of them to an extension actuator (Actuator II in Table S1) and monitor the pressure evolution both at the inlet and inside the actuator using two pressure sensors (MPXV7025DP, Freescale Semiconductor Inc - Fig. S8). During the tests the deformation of the actuator is also recorded by a high-resolution camera (SONY EX100V) at a frame rate of 30 fps. The recorded input/output pressure evolution for each valve is reported in Fig. 1 of the main text.

2.2. Characterization of the kirigami actuators.

As part of this study we also experimentally characterize the pressure-volume evolution of each kirigami actuator using the setup shown in Fig. S9A. To decouple the effect of the geometry of the actuators from that of the compressibility of the fluid, we determine the pressure-volume curve by inflating the actuators with water. Further, to eliminate the influence of gravity, we submerge the entire actuator in a water tank. In all our tests, we first fill the actuator with the amount of water corresponding to the initial volume of the cavity. Then, we use a syringe pump (Pump 33DS, Harvard Apparatus) to displace an additional volume of water $\Delta V$ into the balloons at 20 mL/min (ensuring quasi-static conditions) and record the pressure using a pressure sensor (MPXV7025DP, Freescale Semiconductor Inc). Since as part of this study we use the kirigami actuators in an unconstrained environment for the robotic arm
and constrained within a tube for the climbing robot, we test them both in unconstrained and constrained conditions. More specifically, we test Actuator I and II under unconstrained conditions and find a linear dependence of the pressure from the supplied volume (Fig. S9B). Further, we test Actuator II and III when placed in a tube with diameter of 35 mm (identical to the tube used to test the climbing robot). As shown in Fig. S9C, in this case we find a highly non-linear pressure-volume curve, as the pressure abruptly increases when the actuators get in contact with the tube.

**Fig. S9.** Pressure-volume characterization of the kirigami actuators. (A) Schematic of the test setup used to characterize the pressure-volume relations of actuators. (1) Syringe pump. (2) Pressure sensor. (3) Water tank. (4) Actuator. (B) Pressure-volume relations recorded for Actuators I and II when tested in unconstrained conditions. (C) Pressure-volume relations recorded for Actuators II and III when tested in constrained conditions.
2.3. Robotic Arm. To characterize the deformation of our robotic arm, we monitor with a high-resolution camera (SONY RX100V) recording at a frame rate of 30 fps (i) 17 black circular markers uniformly placed along the length of the kirigami actuators and (ii) a white circular marker placed at the end of an acrylic rod connected to the bottom cap. We extract the coordinates of all markers using an open-source digital image correlation and tracking package (3). We then use the coordinates of the white marker to determine the trajectory followed by the robotic arm. Further, we determine the radius $R$ of the circle that best fits the black markers via a direct least-square algorithm (4, 5) and calculate the average curvature of the two actuators as

$$\kappa = 1/R.$$  \[S1\]

Finally, we note that during the tests the evolution of the pressure inside the actuators is monitored by pressure sensors (MPXV7025DP, Freescale Semiconductor Inc).

In Figs. S16 and S17 we consider the robotic arm with a viscous valve and one-way viscous valve, respectively, and report the recorded tip trajectory (left), pressure evolution in both actuators (middle) and curvature evolution of both actuators (right) for different pressure inputs.
2.4. Tube climbing robot. To test the climbing robot, we place the system inside a vertically oriented acrylic tube with inner diameter of 35 mm and supply many pressure pulses, while recording its motion using a high-resolution camera (SONY RX100V).

Further, we conduct additional tests to characterize both the frictional force between the expanding actuator and the tube and the axial force exerted by the extending actuator interaction between the robot and the tube, as well as the extension force generate by the extending actuator.

As shown in Fig. S10A, to characterize the frictional force, we place an expanding actuator inside a vertically oriented acrylic tube (with inner diameter of 35 mm) and connect it to a motorized translation stage (LTS300/M, ThorLabs) via a nylon thread. During our tests, we inflate the actuator by supplying different amount of pressure (0 – 6 kPa) and pull the actuator upward using the linear stage with speed of 0.2 mm/s. The friction force between the tube and actuator is measured using a 10 lb load cell (LSB200 Miniature S-Beam Jr. Load cell, FUTEK Advanced Sensor Technology, Inc.). The recorded evolution of the friction as a function of applied pressure is reported in Fig. 3C of the main text.

To measure the axial force generated by the extending actuator, we use the same setup as that used to measure the frictional force except that we use screws to connect the load cell to the linear stage and the actuator(Fig. S10B). During our tests, we fix the movement of the linear stage and increase the pressure inside the actuator gradually and record the axial force as a function of the pressure using the load cell. The recorded evolution of the force as a function of applied pressure is reported in Fig. 3C of the main text.

![Fig. S10. Schematics of the experimental setups used to measure frictional and extension forces. (A) Setup for measuring friction. (B) Setup for measuring extension force.](image-url)
2.5. Rolling robot. To actuate our rolling robot we connect it to our pressure control system (Fig. S8) and supply the pressure profile shown in Fig. S11. Note that the input pressure for each cycle consists of 2 pressure pulses:

- **Pulse 1**: the first pressure pulse \( p_{\text{input}} = 15 \text{ kPa for } t_{\text{input}} = 3 \text{ s} \) is provided to rapidly inflate the chambers connected directly to the pressure supplier (snapshot 2 in Fig. 4C of main text). This high pressure pulse accelerate the inflation of the chambers connected directly to the pressure source but has negligible effect on the chambers attached to the viscous valves.

- **Pulse 2**: a moderate pressure pulse \( p_{\text{input}} = 10 \text{ kPa for } t_{\text{input}} = 67 \text{ s} \) then follows the high pressure pulse to gradually inflate the chambers attached to the viscous valves. Note that after supplying this second pulse all chambers are inflated (snapshot 4 in Fig. 4C of main text).

When the input pressure is removed, the chambers with no viscous valve deflate instantaneously, while the ones with viscous valves keep the inflated state temporarily and push the robot to rotate further (snapshot 5 in Fig. 4C of main text).

To change the rolling direction of the robot we apply a negative pressure burst with magnitude large enough (~25 kPa) to change the position of the piston and snap the arch at the same time.

![Fig. S11. Pressure supplied to the rolling robot for bidirectional locomotion.](image-url)
3. Modeling

To get a better understanding of the behavior of the proposed valves, we use numerical analyses. For the viscous valves, we simplify the Navier–Stokes equations to calculate the pressure drop. For the hysteretic and bistable valves, we conduct Finite Element (FE) simulations to characterize the mechanical response of the arches.

3.1. Viscous valves. Our viscous valves comprise narrow tubes to provide a transient pressure difference between adjacent actuators. To predict their response, we use a numerical model recently proposed by Vasios et al (6). More specifically, we focus on the \([i]\)-th valve in the actuator that comprises a narrow tube of length \(l_{\text{tube},i}\) and diameter \(d_{\text{tube},i}\) and assume that

- the tube is rigid and not deformed by the flow;
- the head losses due to friction at the connections between the tube and the actuators can be captured by adjusting its length to \(l_{\text{eq},i}\);
- the flow is incompressible and laminar in the narrow tube;
- the fluid velocity has the form

\[
\mathbf{u} = -\frac{8}{\pi d_{\text{tube},i}^2} \frac{d\tilde{v}_i}{dt} \left[ \left( \frac{r}{R_i} \right)^2 - 1 \right] \mathbf{e}_z, \tag{S2}
\]

where \(\tilde{v}_i = \int_0^t \int_0^{l_{\text{tube},i}/2} \mathbf{u} \cdot \mathbf{e}_z 2\pi r dr dt\) denotes the amount of fluid exchanged through the \([i]\)-th tube up to time \(t\) and \(\mathbf{e}_z\) identifies the tangent vector to the tube;
- the inertia term is negligible, since the tubes are narrow (\(l_{\text{tube},i} >> d_{\text{tube},i}\)).

Under these assumptions, integration of the Navier–Stokes equations over the volume of the tube yields

\[
\frac{d\tilde{V}_i}{dT} + \xi_i (P_i - P_{i-1}) = 0, \quad \text{for} \quad i = 1, \ldots, N \tag{S3}
\]

with

\[
\xi_i = \frac{\pi Gd_{\text{tube},i}^4 t_{\text{max}}}{128 \mu v_0 l_{\text{eq},i}}, \tag{S4}
\]

where \(\tilde{V}_i = \tilde{v}_i/v_0\) is the normalized volumetric flow through the \([i]\)-th tube (\(v_0\) denoting the volume of the largest actuator in the system), \(T = t/t_{\text{max}}\) is the normalized time (\(t_{\text{max}}\) denoting the response time of the largest actuator).
time of the system) and \( P_i = p_i / G \) is the normalized pressure in the \([i]\)-th actuator (\( p_i \) being the pressure inside the \([i]\)-th actuator and \( G \) being the shear modulus of the material used to fabricate the actuator). Note that \( 1 / \xi_i \) expresses an equivalent resistance that the \([i]\)-th valve imposes to fluid flow; whereas \( \xi_i \) is large, high flow rates \( dV_i / dT \) are achieved for relatively low pressure differences, whereas when \( \xi_i \) is small, the opposite is true.

Next, we connect the amount of fluid exchanged through the valves to the amount of fluid present in the chambers of the actuators. When an incompressible fluid such as water is used to inflate the system, the normalized change in volume for the \([i]\)-th actuator, \( \Delta V_i = \Delta v_i / v_0 \), can be expressed in terms of the volumetric flows exchanged through the two tubes connected to it as

\[
\Delta V_i = \bar{V}_i - \bar{V}_{i+1}. \tag{S5}
\]

Differently, when, as in this study, we use a compressible fluid (air) to inflate the robotic system, we focus on the number of moles of fluid exchanged through the valves. Specifically, the change in number of moles of air for the \([i]\)-th actuator, \( \Delta n_i \), can be determined as

\[
\Delta n_i = \bar{n}_i - \bar{n}_{i+1}, \tag{S6}
\]

where \( \bar{n}_i \) denotes the number of moles of air exchanged through the \([i]\)-th valve. Further, the total number of moles of air in the \([i]\)-th actuator at time \( T \) can be expressed as

\[
n_i(T) = n_i(T = 0) + \int_0^T \frac{d\Delta n_i}{dT} dT = n_i(T = 0) + \int_0^T \frac{d(\bar{n}_i - \bar{n}_{i+1})}{dT} dT. \tag{S7}
\]

Note that, by introducing the ideal gas law \( n_i \) and \( \bar{n}_i \) can be written as

\[
n_i = \frac{P_i V_i}{R_g \Theta_i}, \quad \bar{n}_i = \frac{\bar{P}_i \bar{V}_i}{R_g \bar{\Theta}_i}, \tag{S8}
\]

where \( V_i \) denotes the volume for the \([i]\)-th actuator, \( R_g \) is the ideal gas constant and \( \Theta_i \) is the temperature. Moreover,

\[
\bar{P}_i(T) = \begin{cases} P_{i-1} & \text{for } \frac{d\bar{V}_i}{dT} \geq 0, \\ P_i & \text{for } \frac{d\bar{V}_i}{dT} < 0, \end{cases}
\]

and

\[
\bar{\Theta}_i(T) = \begin{cases} \Theta_{i-1} & \text{for } \frac{d\bar{V}_i}{dT} \geq 0, \\ \Theta_i & \text{for } \frac{d\bar{V}_i}{dT} < 0. \end{cases}
\]

Note that if the air flows from the \([i-1]\)-th actuator to \([i]\)-th actuator, the pressure and temperature inside the \([i-1]\)-th actuator should be used to calculate \( \bar{n}_i \), whereas when the air flows from the \([i]\)-th actuator to \([i-1]\)-th actuator, \( \bar{n}_i \) is determined based on the pressure and temperature inside the \([i]\)-th actuator.

If we assume the actuation to be an isothermal process (i.e. we assume that the temperature of the air inside the actuators is constant), substitution of Eqs. (S8) into Eq. (S7) yields

\[
P_i(T)V_i(T) = P_i(0)V_i(0) + \int_0^T \bar{P}_i(T) \frac{d\bar{V}_i(T)}{dT} dT - \int_0^T \bar{P}_{i+1}(T) \frac{d\bar{V}_{i+1}(T)}{dT} dT, \tag{S9}
\]

where \( V_i(0) \) is the initial volume of the \([i]\)-th actuator and \( P_i(0) \) is the initial pressure inside the \([i]\)-th actuator (i.e. atmospheric pressure \( P_i(0) = 101.3 \text{ kPa} \)). Finally, by substituting Eq. (S3) into
We obtain

\[ P_i(T) V_i(T) = P_i(0) V_i(0) - \int_0^T \xi_i \bar{P}_i(T) [P_i(T) - P_{i-1}(T)] \, dT + \int_0^T \xi_{i+1} \left[ \bar{P}_{i+1}(T)(P_{i+1}(T) - P_i(T)) \right] \, dT, \tag{S10} \]

For a system comprising \( N \) fluidic actuators interconnected via viscous valves Eq. (S10) results in a system of \( N \) coupled differential equations, which given a pressure-volume relationship for the actuators that can be numerically solved to determine the normalized change in volume for the \( i \)-th actuator as a function of time. Finally, we note that for the first and last tube in the array Eq. (S10) needs to be modified as

\[ P_1(T) V_1(T) = P_1(0) V_1(0) - \int_0^T \xi_1 \bar{P}_1(T) [P_1(T) - P_{\text{input}}(T)] \, dT + \int_0^T \xi_2 \left[ \bar{P}_2(T)(P_2(T) - P_1(T)) \right] \, dT, \tag{S11} \]

and

\[ P_N(T) V_N(T) = P_N(0) V_N(0) - \int_0^T \xi_N \bar{P}_N(T) [P_N(T) - P_{N-1}(T)] \, dT, \tag{S12} \]

to account for the pressure input and the end of the array.

**Individual valves.** For the simple setup considered in Fig. 1D of the main text (comprising a rigid chamber and an extension kirigami actuator separated by a viscous valve), Eq. (S10) specializes to

\[ P(T) V(T) = P(0) V(0) - \int_0^T \xi \bar{P}(T)(P(T) - P_{\text{input}}(T)) \, dT, \tag{S13} \]

where \( P \) and \( \bar{V} \) denote the volume and pressure of the actuator and \( P_{\text{input}}(t) \) is the normalized input pressure supplied to the rigid chamber

\[ P_{\text{input}}(T) = \begin{cases} \frac{P_{\text{input}}}{G} & \text{for } T \leq T_{\text{input}}, \\ 0 & \text{for } T > T_{\text{input}}. \end{cases} \tag{S14} \]

Note \( P \) and \( V \) are not independent, as they follow the experimentally measured pressure-volume relationship shown in Fig. S9. To estimate \( \xi \) for a given valve we compare the time evolution of pressure measured in our experiments with the numerical prediction obtained by numerically solving Eq. (S13) with the pressure-volume relationship reported in Fig. S9B (Actuator II). We find excellent agreement between experimental results and numerical predictions when choosing (see Fig. S13)

- \( \xi = 14.2 \) for a viscous valve with \( d_{\text{tube}} = 0.21 \) mm;
- \( \xi = 14.2 \) for a one-way viscous valve with \( d_{\text{tube}} = 0.21 \) mm and flow in the forward direction
- \( \xi = 716.2 \) for a one-way viscous valve with \( d_{\text{tube}} = 0.21 \) mm and flow in the backward direction

Note that these values for \( \xi \) are also summarized in Table S2.

**Robotic arm.** For the robotic arm considered in Fig. 2 of the main text (which comprises two bending kirigami actuators connected through a viscous valve with \( d_{\text{tube}} = 0.21 \) mm), Eq. (S10) specializes to
One-way viscous valve

\[ P_1(T)V_1(T) = P_1(0)V_1(0) - \int_0^T \xi_1 \bar{P}_1(T)(P_1(T) - P_{input}(T))dT + \int_0^T \xi_2 \bar{P}_2(T)(P_2(T) - P_1(T))dT, \]

\[ P_2(T)V_2(T) = P_2(0)V_2(0) - \int_0^T \xi_2 \bar{P}_2(T)(P_2(T) - P_1(T))dT, \]

where the subscripts 1 and 2 refer to the top and bottom actuator, respectively. Further, \( \xi_1 \) and \( \xi_2 \) denote the equivalent resistance encountered by the fluid between the input source and the top actuator and the equivalent resistance imposed by the viscous valve, respectively. Note that the values for \( \xi_2 \) are reported in Table S2, whereas for the connection between the input source and the first actuator we use \( \xi_1 = 472.7 \). Finally, to connect \( P \) and \( V \), we use the experimentally measured pressure-volume relationship shown in Fig. S9B (Actuator I). As shown, in Figs. S16 and S17 the model nicely capture the response of the arm with both a viscous and one-way viscous valve for a wide range of pressure inputs.

Climbing robot. For the climbing robot considered in Fig. 3 of the main text (which comprises three bending kirigami actuators connected through a viscous valve with \( d_{tube} = 0.21 \) mm and a one-way viscous valve with \( d_{tube} = 0.21 \) mm), Eq. (S10) specializes to

\[ P_1(T)V_1(T) = P_1(0)V_1(0) - \int_0^T \xi_1 \bar{P}_1(T)(P_1(T) - P_{input}(T))dT + \int_0^T \xi_2 \bar{P}_2(T)(P_2(T) - P_1(T))dT, \]

\[ P_2(T)V_2(T) = P_2(0)V_2(0) - \int_0^T \xi_2 \bar{P}_2(T)(P_2(T) - P_1(T))dT + \int_0^T \xi_3 \bar{P}_3(T)(P_3(T) - P_2(T))dT, \]

where the subscripts 1, 2 and 3 refer to the bottom, middle and top actuators. Further, \( \xi_1 \) is the equivalent resistance between the input source and the bottom actuator, \( \xi_2 \) denotes the equivalent resistances of the one-way viscous valve and \( \xi_3 \) represent the equivalent resistances of the viscous valve. Note that the values for \( \xi_2 \) and \( \xi_3 \) are reported in Table S2, whereas for the connection

<table>
<thead>
<tr>
<th></th>
<th>Viscous valve ((d_{tube} = 0.21 ) mm)</th>
<th>One-way viscous valve (d_{tube} = 0.21 ) mm [forward direction]</th>
<th>One-way viscous valve (d_{tube} = 0.21 ) mm [backward direction]</th>
</tr>
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<tbody>
<tr>
<td>( \xi_i )</td>
<td>14.2</td>
<td>14.2</td>
<td>716.2</td>
</tr>
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Table S2. Equivalent conductance of the viscous valves.
between the input source and the first actuator we use $\xi_1 = 472.7$. Finally, to connect $P$ and $V$, we use the experimentally measured pressure-volume relationship shown in Fig. S9. As shown in Fig. S20, we find that the model (dashed lines) agree well with the experimental measurements (solid lines) for all considered pressure inputs.

3.2. Snapping arch. To identify the geometric features resulting either in a snapping or in a bistable arch, we perform Finite Element (FE) analyses using the commercial package ABAQUS 2019/Standard. In our analyses we consider plates with width $w_{\text{plate}} = 5$ mm, length $l_{\text{plate}} \in [17, 19.5]$ mm, thickness $t_{\text{plate}}$ equal to either 0.05 mm or 0.075 mm and mounting angle $\theta_{\text{plate}} \in [0^\circ, 60^\circ]$. We discretize the plate using four-node general-purpose shell elements with reduced integration and hourglass control (S4R element type). Further, since plasticity has little effect on the observed phenomena, we capture the material behavior using a linear elastic material model (with Young’s modulus $E = 170$ GPa and Poisson’s ratio $\nu = 0.3$). We then simulate the response of the arch conducting non-linear static simulations under displacement control and dynamic implicit simulations under load control. The simulations consist of two steps

- **Step 1:** We buckle a flat plate to an arch by applying an in-plane compression displacement $d_{\text{in}} = (l_{\text{plate}} - w)/2$ ($w = 16$ mm being the distance between the two slits of the holder - Fig. S14B) and a rotation angle $\theta_{\text{plate}}$ to both end of the arch.

- **Step 2:** We fix the displacement and rotation at both ends of the arch. For displacement control simulations we apply an out-of-plane displacement $d_{\text{out}}$ at the center of the arch (see Fig. S14A) and record the reaction force $F$. For the load control simulations, we apply a force $F$ at the center of the arch (see Fig. S14B) and record the displacement $d_{\text{out}}$. Note that the force $F$ can be expressed as a function of the applied pressure as

$$F = p\pi r^2,$$

where $r$ is the radius of the chamber.

In Fig. S14C, we report the numerically predicted evolution of the equivalent pressure $p$ as a function of the out-of-plane displacement $d_{\text{out}}$ for an arch with $\theta_{\text{plate}} = 45^\circ$, $l_{\text{plate}} = 17.5$ mm and $t_{\text{plate}} = 0.075$ mm. The continuous line represents the FE results of the arch under displacement control (by applying a displacement $d_{\text{out}}$ at the center of the arch). The non-monotonic behavior of the arch is clearly visible. However, in our mechanical valves the flow pressurizes the piston, which transfers the force to the central point of the arch. Hence, the boundary conditions of the experiments are different from those of the simulations under displacement control. To quantify the effect of such boundary conditions, we also report the response obtained from our load control simulations (black dashed curves in Fig. S14C). In this case we can see that the pressure-displacement relation of the arch exhibits a linear regime at the beginning of the loading step. Then, as the pressure increases to a certain value $p_{\text{on}}^*$, the central point of the arch jumps from positions I to II (characterized by a plateau on the curve) and, in turn, changes the position of the piston and switches on the valve. Furthermore, when we reduce the pressure, the arch can snap back at pressure $p_{\text{off}}^*$ from positions III to IV and switch off the valve.

It is worth noticing that the model results have to be corrected by taking into account the contribution of the friction force between the piston and the chamber. We measure this force to be $F_{\text{friction}} = 2.2$ N and always opposed to the moving direction of the piston. Therefore the real
Fig. S14. FE simulations for the hysteretic and bistable valves. (A) Experimental snapshots of a bistable valve in the closed and open states. (B) Schematic of a 3D printed holder for the plate. The mechanical behavior of the valve can be tuned by changing the mounting angle ($\theta_{\text{plate}}$), length ($l_{\text{plate}}$), and thickness ($t_{\text{plate}}$) of the plate (we fix the width of the plate $w_{\text{plate}} = 5\, \text{mm}$ throughout this study). (C) Mechanical response of an hysteretic valve with $\theta_{\text{plate}} = 45^\circ$, $l_{\text{plate}} = 17.5\, \text{mm}$ and $t_{\text{plate}} = 0.075\, \text{mm}$. Continuous red line and black dashed lines represent the FE results of the plate under displacement control and force control, respectively. Red dashed lines represent the behavior of the valve with consideration of the friction between the piston and the chamber. The on/off pressures of the valve are $p_{\text{on}} = 20.8\, \text{kPa}$ and $p_{\text{off}} = 2.1\, \text{kPa}$, respectively. (D,E) Phase diagrams of ($p_{\text{on}}$) and ($p_{\text{off}}$) for a valve with $t_{\text{plate}} = 0.075\, \text{mm}$ as a function of $\theta_{\text{plate}}$ and $l_{\text{plate}}$. (F) Mechanical response of a bistable valve with $\theta_{\text{plate}} = 0^\circ$, $l_{\text{plate}} = 17.5\, \text{mm}$ and $t_{\text{plate}} = 0.05\, \text{mm}$. Continuous orange line and black dashed lines represent the FE results of the plate under displacement control and force control, respectively. Orange dashed lines represent the behavior of the valve with consideration of the friction between the piston and the chamber. The on/off pressures of the valve are $p_{\text{on}} = 16.3\, \text{kPa}$ and $p_{\text{off}} = -16.3\, \text{kPa}$, respectively. (G,H) Phase diagrams of ($p_{\text{on}}$) and ($p_{\text{off}}$) for a valve with $t_{\text{plate}} = 0.05\, \text{mm}$ as a function of $\theta_{\text{plate}}$ and $l_{\text{plate}}$.

The response of the valve can be obtained by shifting the black dashed line up (for the loading curve) or down (for the unloading curve) by the amount of the equivalent pressure generated by the friction force (see red dashed lines in Fig. S14C). As a result, the corrected on/off pressure of the valve reads

$$p_{\text{on}} = p^*_\text{on} + \frac{F_{\text{friction}}}{\pi r^2}, \quad p_{\text{off}} = p^*_\text{off} - \frac{F_{\text{friction}}}{\pi r^2}. \quad [S18]$$

In order to design a valve with a prescribed on/off pressure, in Figs. S14D and E we report the phase diagrams of $p_{\text{on}}$ and $p_{\text{off}}$ for a valve with $t_{\text{plate}} = 0.075\, \text{mm}$ as a function of the mounting
angle $\theta_{plate}$ and length $l_{plate}$. Instructed by the model, we choose $\theta_{plate} = 45^{\circ}$ and $l_{plate} = 17.5$ mm as design parameters, which enables the on/off switch of the valve at 20.8 kPa and 2.1 kPa, respectively.

For the bistable valve, we reduce the thickness of the plate to $t_{plate} = 0.05$ mm in order to operate the valve at around 20 kPa. Similarly, we report the response of the bistable valve (with $\theta_{plate} = 0^{\circ}$ and $l_{plate} = 17.5$ mm ) in Fig. S14F and the phase diagrams for $p_{on}$ and $p_{off}$ for a valve with thickness $t_{plate} = 0.05$ mm in Figs. S14G and H, respectively. For the rolling robot, we choose $\theta_{plate} = 0^{\circ}$ and $l_{plate} = 17.5$ mm as design parameters of the bistable valve, which enable the on/off switch of the valve at 16.3 kPa and $-16.3$ kPa, respectively.

Please note that under load control the simulation of the snapping arch beyond point I or III often incurs in convergence problems (Fig. S14C), providing incomplete information about the response of the arch. However, the local maximum and minimum pressures for the displacement control curves are identical to those for the load control curves. Therefore the $p^*_on$ and $p^*_off$ values can be extracted from the displacement control simulations. In turn, we can calculate $p_{on}$ and $p_{off}$ for the phase diagrams by correcting these results to take into account friction.

**Fig. S15.** Effect of the equivalent conductance $\xi$ on the response of our viscous valves. (A) Numerically predicted pressure evolution at the inlet and outlet of a viscous valve characterized by $\xi = 14.2$, 28.4 and 42.6 when connected to the extension actuator considered in Fig. 1c. (B) Numerically predicted delay time between the input and output (i.e. the time that it takes $p_{output}$ to reach $p_{input}$) as a function of $\xi$. 

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4. Additional Results

Fig. S16. Robotic arm comprising two bending actuators and a viscous valve. Multiple trajectories of the robotic arm are realized upon application of (A) a short rectangular pressure pulse ($p_{\text{input}} = 20$ kPa for $t_{\text{input}} = 4$ s), (B) a long rectangular pressure pulse ($p_{\text{input}} = 20$ kPa for $t_{\text{input}} = 26$ s) and (C) a gradually varied input pressure profile. Our model can accurately predict the pressure and curvature evolution of each actuator.
Fig. S17. Robotic arm comprising two bending actuators and a one-way viscous valve. Multiple trajectories of the robotic arm are realized upon application of (A) a short rectangular pressure pulse ($p_{\text{input}} = 20$ kPa for $t_{\text{input}} = 4$ s), (B) a long rectangular pressure pulse ($p_{\text{input}} = 20$ kPa for $t_{\text{input}} = 26$ s) and (C) a gradually varied input pressure profile. Our model can accurately predict the pressure and curvature evolution of each actuator.
Fig. S18. Numerical results of a robotic arm comprising three bending actuators and two one-way viscous valves. (A) Schematic of the robotic arm. (B-D) Numerical results for the robotic arm upon application of (B) a short rectangular pressure pulse ($P_{\text{input}} = 20$ kPa for $t_{\text{input}} = 4$ s), (C) a long rectangular pressure pulse ($P_{\text{input}} = 20$ kPa for $t_{\text{input}} = 50$ s), and (D) a gradually varied input pressure profile.
Fig. S19. Numerical results of a robotic arm comprising three bending actuators and two one-way viscous valves. (A) Schematic of the robotic arm. (B-D) Numerical results for the robotic arm upon application of (B) a short rectangular pressure pulse ($p_{\text{input}} = 20$ kPa for $t_{\text{input}} = 4$ s), (C) a long rectangular pressure pulse ($p_{\text{input}} = 20$ kPa for $t_{\text{input}} = 40$ s) and (D) a gradually varied input pressure profile. Note that this robotic arm is different from the one in Fig. S18 because of the flipped configurations of both viscous valves.
Fig. S20. Pressure evolution and displacements of the climbing robot upon application of a wide range of pressure inputs. (A) The robot falls down at the end of each cycle with $t_{input} = 45$ s. (B) The robot can not move upwards after the first cycle with $t_{input} = 55$ s. (C) The robot climbs up to 16 cm in 30 cycles but can not climb with additional weight (100 g) for $t_{input} = 48$ s. (D) The robot climbs up to 8 cm in 30 cycles with additional weight (100 g) with $t_{input} = 50$ s.
Fig. S21. Phase diagram of the minimum pressure inside the climbing robot’s top actuator when a hysteretic valve is embedded into it. The embedded hysteretic valve affects the cavity of the top actuator and, in turn, the pressure-volume evolution. The area of the diagram that is bounded by the two dashed lines identifies the input parameters for which the robot will achieve climbing. Guided by our model, we choose $p_{\text{input}} = 48$ s and $p_{\text{input}} = 15$ kPa as the input for the grasping tests.
Fig. S22. Effect of the equivalent conductance of both the pump and the valves on actuation speed. (A) Pressure evolution inside the three actuators for the climbing robot considered in Fig. 3 of the main text. (B) Numerically predicted pressure evolution inside the three actuators for a climbing robot for which the equivalent conductance $\xi$ of both the pump and the valves is increased by 10 times.
Movie S1. Characterization of the mechanical valves.

To demonstrate the capabilities of our mechanical valves, we connect each valve to an extending actuator and compare the pressure evolution at the inlet and outlet of the valve.

Movie S2. Robotic arm with different trajectories.

The robotic arm consists of two bending actuators and a viscous valve (or one-way viscous valve). It is capable of achieving multiple trajectories with a single pressure input.

Movie S3. Tube climbing robot.

The climbing robot consists of two expanding actuators, an extending actuator, a viscous valve and a one-way viscous valve. It can climb inside a pipe carrying two times its own weight. When a threshold valve and a gripper are integrated in the robot, the robot can also grasp an object and pull it down. All the movements are achievable with a single pressure input.

Movie S4. Rolling robot.

The hexagonal rolling robot comprises 12 inflatable chambers and six viscous valves arranged into two circuits. A bistable valve activates one circuit at a time, enabling the robot to successfully navigate in both directions with a single input.
References


